A complete classification of homogeneous plane continua

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joint with L. G. Oversteegen

Nipissing University

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All spaces are metric.

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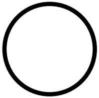

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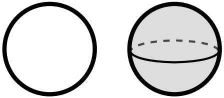

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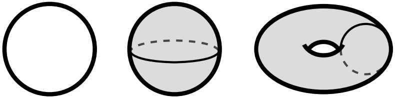

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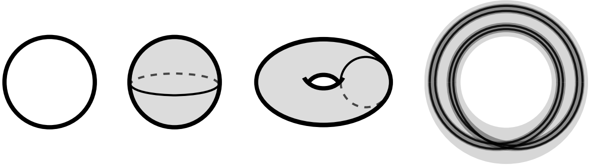

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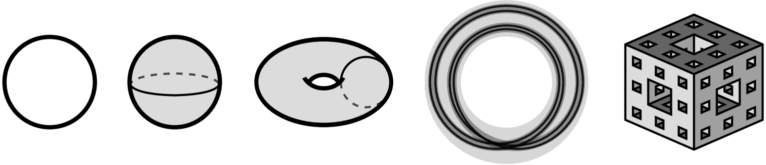

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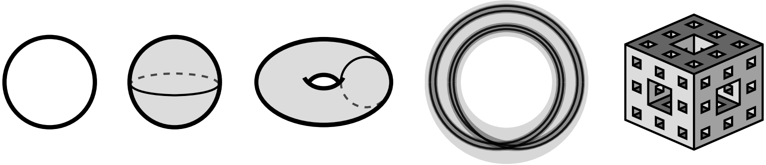

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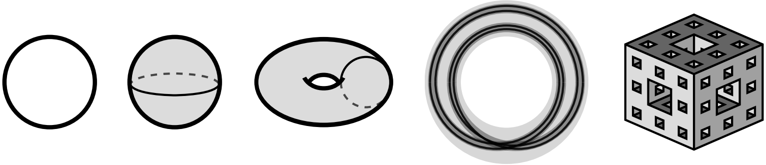

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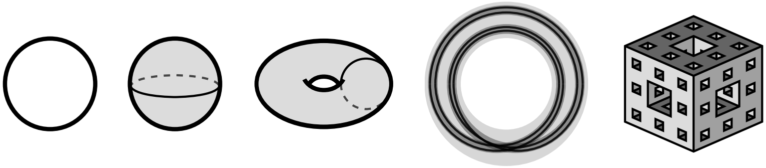

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Question (Knaster-Kuratowski 1920)

Is the circle the only non-degenerate homogeneous continuum in $\mathbb{R}^2$?

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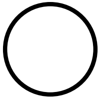

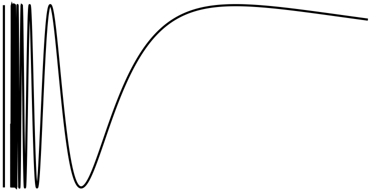

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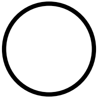
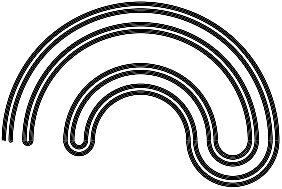
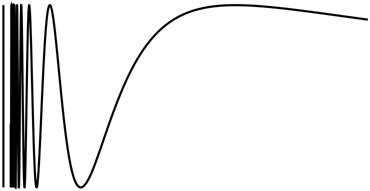


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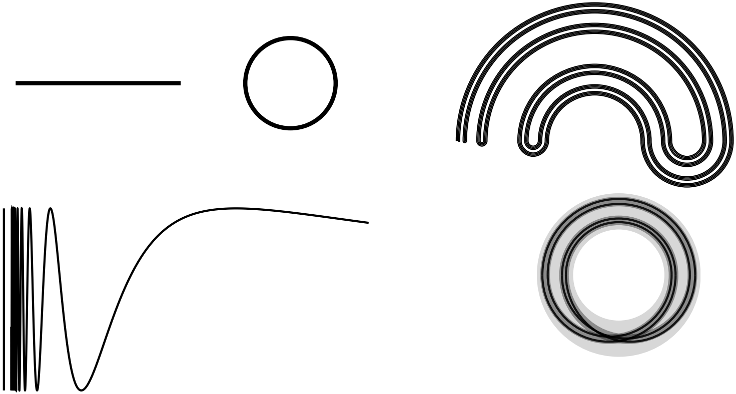

Hereditarily indecomposable $\equiv$ every subcontinuum is indecomposable

The pseudo-arc

Example (Knaster 1922, Moise 1948, Bing 1948)**:** The *pseudo-arc*.

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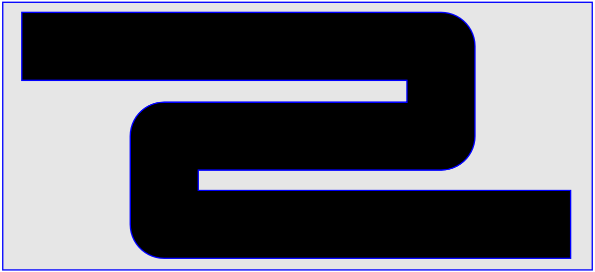

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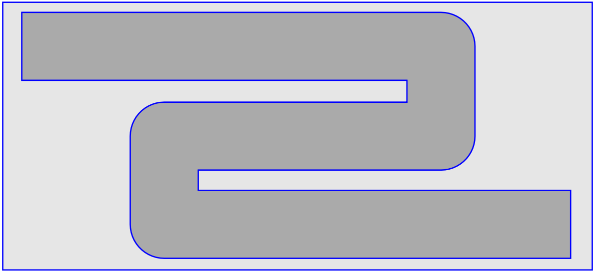

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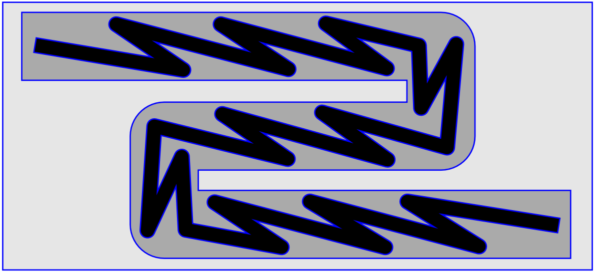

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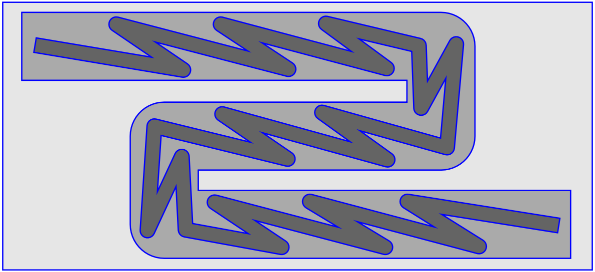

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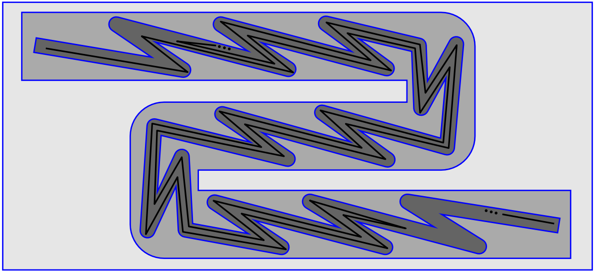

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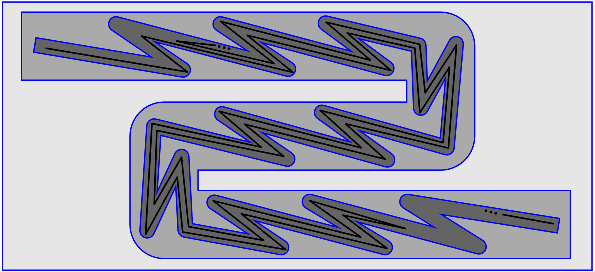

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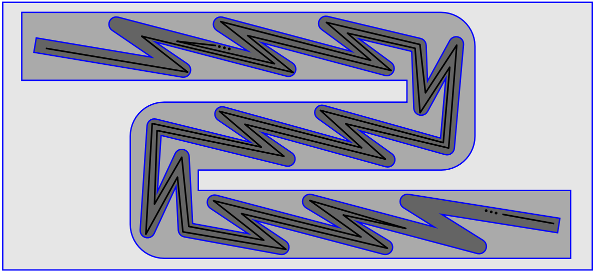

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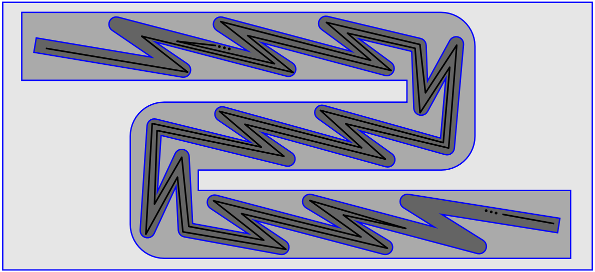

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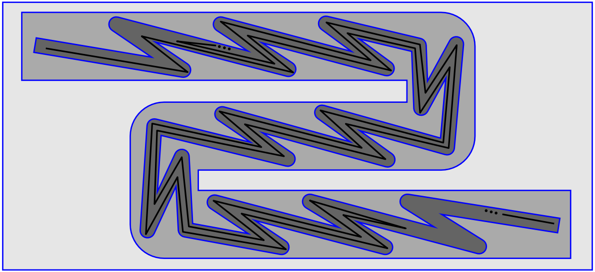

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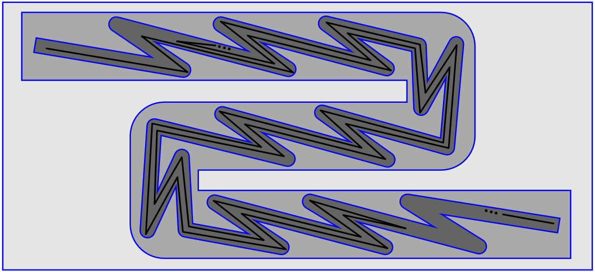

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Theorem (Bing 1951)

$X \approx$ pseudo-arc if and only if X is hereditarily indecomposable and arc-like.

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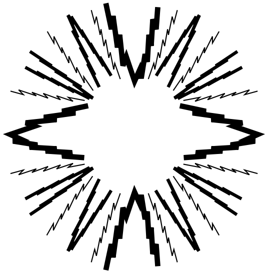

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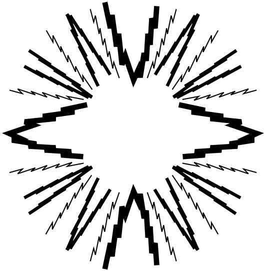

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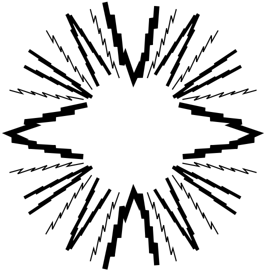

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Theorem (Hagopian 1976)

If $X \subset \mathbb{R}^2$ is indecomposable homogeneous, then X is hereditarily indecomposable.

Homogeneous plane continua and span

X has *span zero* $\equiv$ any continuum Z in $X \times X$ with $\pi_1(Z) \subseteq \pi_2(Z)$ meets $\Delta X = \{(x,x) : x \in X\}$.

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Corollary

If $X \subset \mathbb{R}^2$ is a (non-degenerate) homogeneous continuum, then $X \approx$ the circle, the pseudo-arc, or the circle of pseudo-arcs.

Span and separators

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Theorem

$\forall\, \varepsilon > 0 \quad \exists\, \delta > 0$ such that if T is a tree and $I = [p,q]$ is an arc, with $d_H(T,X) < \delta$ and $d_H(I,X) < \delta$, then the set

$$M = \{(x,y) \in T \times I : d(x,y) < \varepsilon\}$$

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Then $\langle Z_n \rangle$ accumulates on some $Z \subset X \times X$, where $\pi_2(Z) = X$ and $d(x,y) \geq \varepsilon$ for all $(x,y) \in Z$. Thus X has span > 0. □

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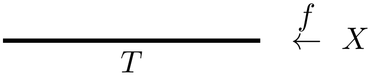

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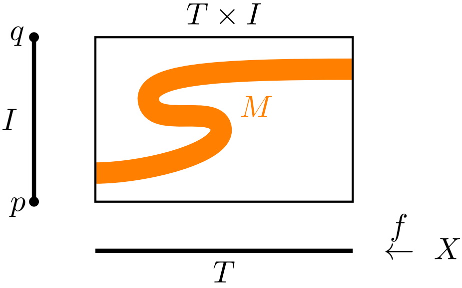

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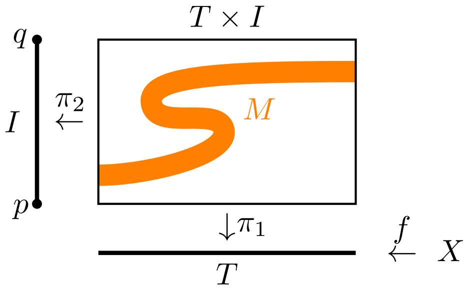

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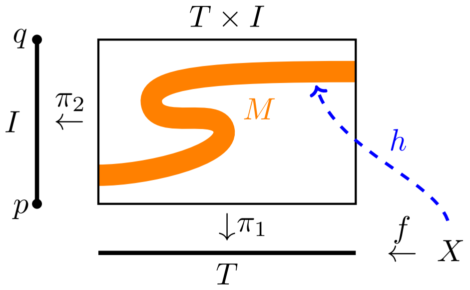

If $\exists\ h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.

Simple folds

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Simple fold on a graph G:

- Subgraphs $G_1, G_2, G_3 \subset G$ such that
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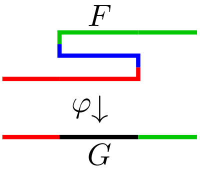

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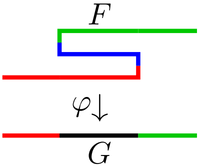

Theorem (cf. Krasinkiewicz-Minc 1977)

X is hereditarily indecomposable if and only if for any map $f : X \to G$ to a graph G, for any simple fold $\varphi : F \to G$, and for any $\varepsilon > 0$, there exists a map $g : X \to F$ such that $\varphi \circ g =_\varepsilon f$.

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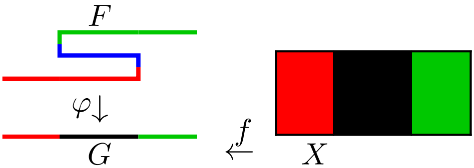

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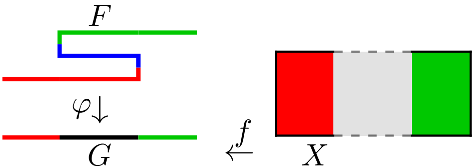

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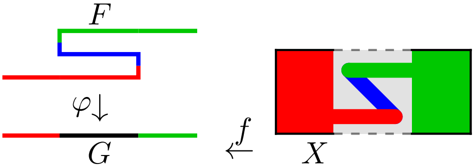

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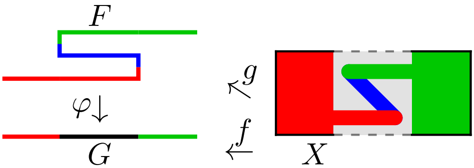

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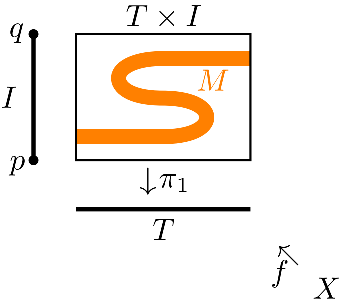

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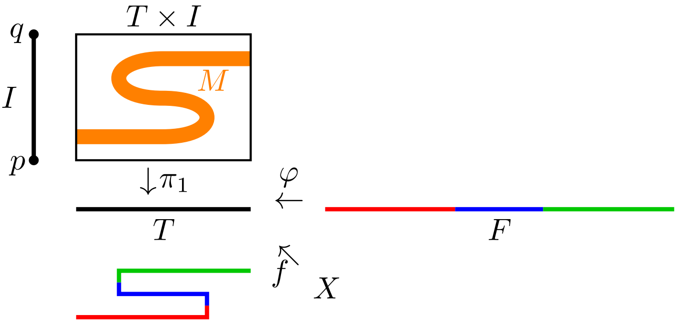

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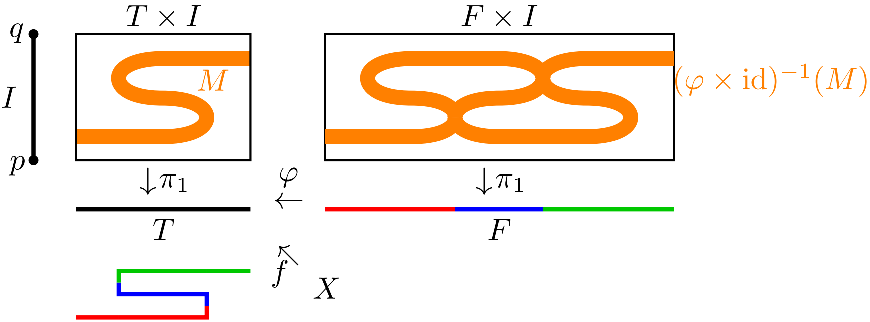

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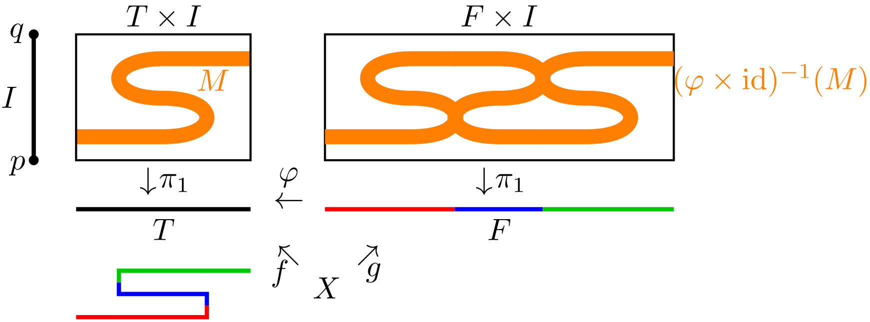

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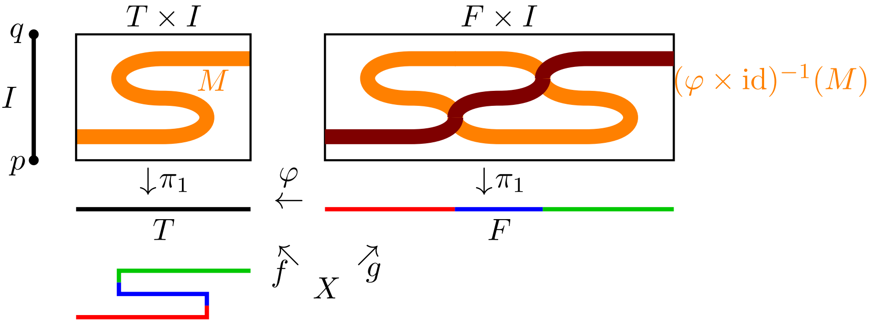

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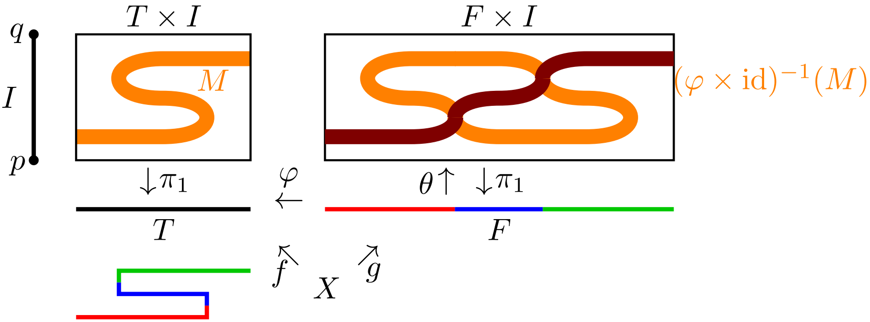

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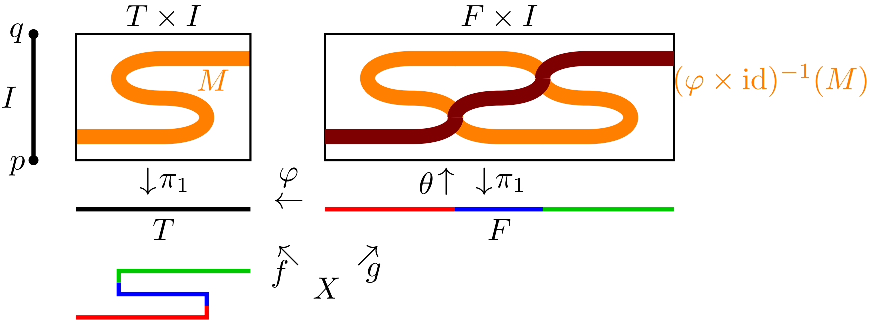

Define $h : X \to M$ by $h = (\varphi \times \mathrm{id}) \circ \theta \circ g$.

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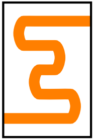

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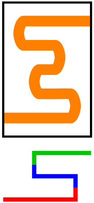

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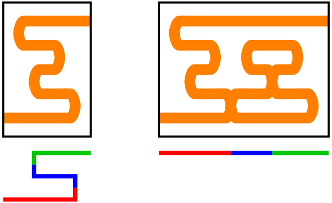

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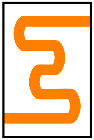

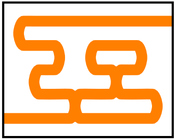

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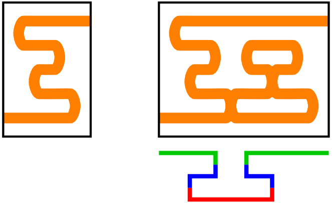

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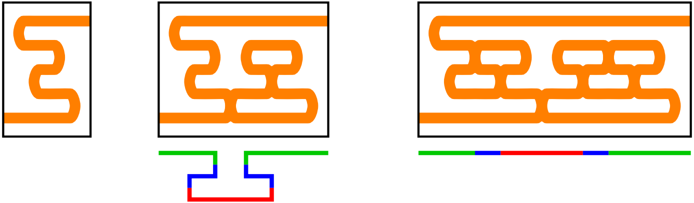

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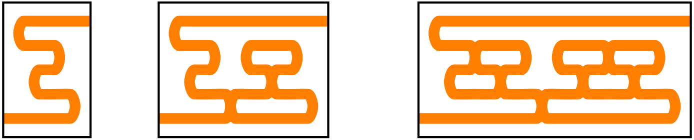

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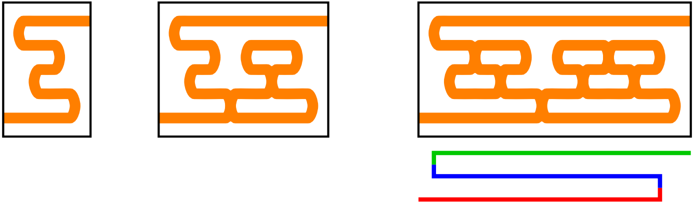

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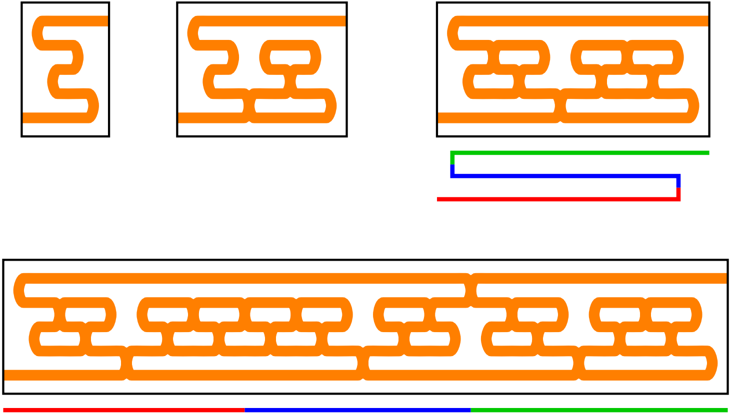

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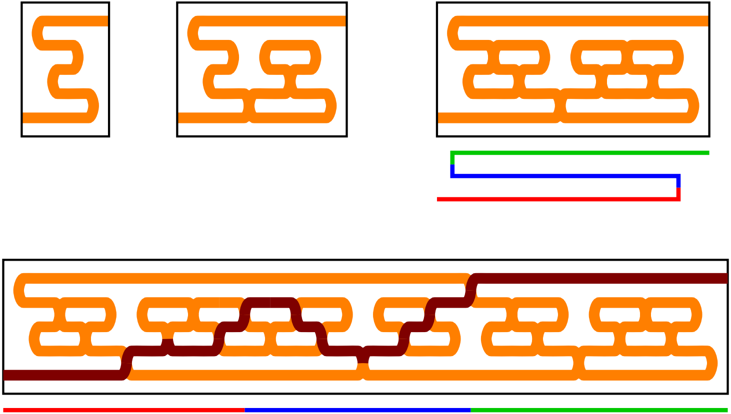

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