Workshop on homogeneous plane continua

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on joint work with L. G. Oversteegen

Nipissing University

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Baylor University

Continua: basics 1

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$\Leftrightarrow$ X is an inverse limit of trees
$\Leftrightarrow$ There is a sequence of trees $\langle T_n \rangle_{n=1}^{\infty}$ in $[0, 1]^{\mathbb{N}}$ and maps $f_n : X \to T_n$ such that $f_n \to \mathrm{id}_X$ (uniformly)

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Theorem (Oversteegen-H 2015)

A continuum is homeomorphic to the pseudo-arc if and only if it is hereditarily indecomposable and has span zero.

Simple folds

Simple fold on a graph G:

- Subgraphs $G_1, G_2, G_3 \subset G$ such that
  - $G_1 \cup G_3 = G$ and $G_1 \cap G_3 = G_2$;
  - $\overline{G_1 \setminus G_2} \cap \overline{G_3 \setminus G_2} = \emptyset$.
- Graph $F = F_1 \cup F_2 \cup F_3$ and map $\varphi : F \to G$ such that
  - $\varphi\restriction_{F_i} : F_i \to G_i$ is a homeomorphism for each $i = 1, 2, 3$;
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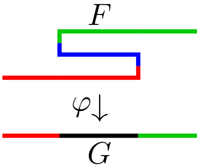

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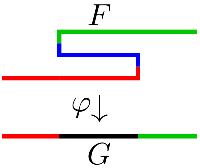

Theorem (cf. Krasinkiewicz-Minc 1977)

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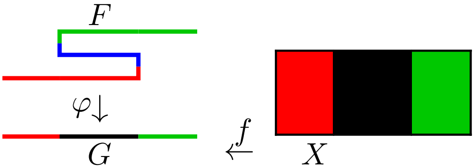

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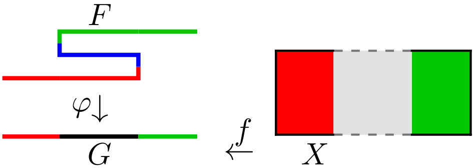

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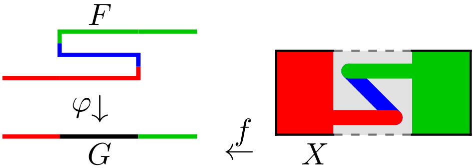

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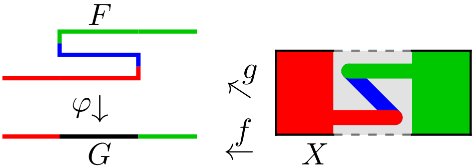

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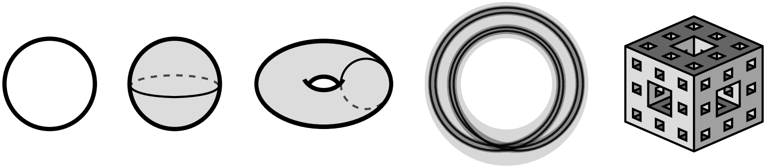

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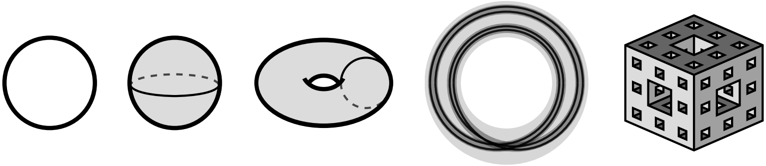

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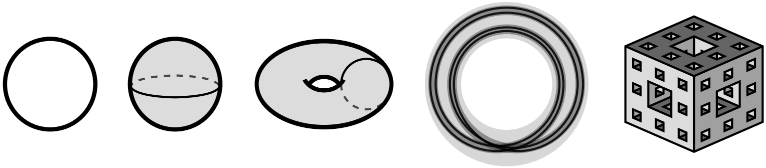

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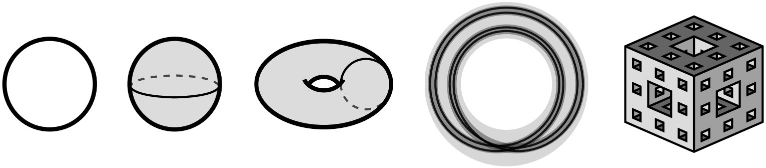

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Answer: **No.** Known examples: circle, pseudo-arc, circle of pseudo-arcs

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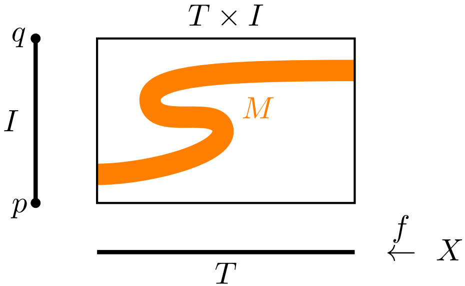

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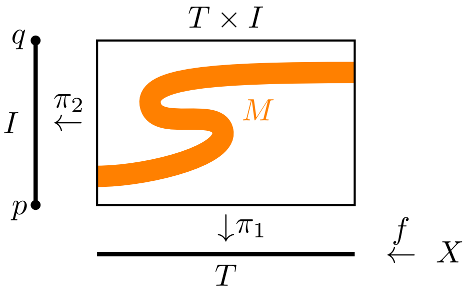

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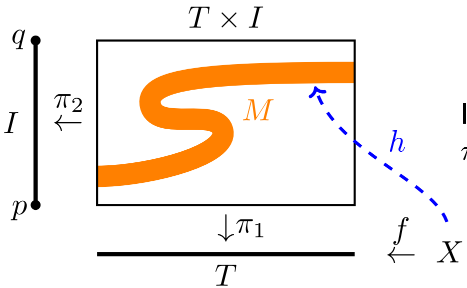

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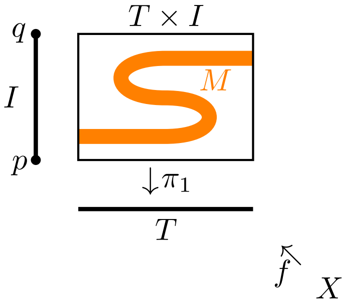

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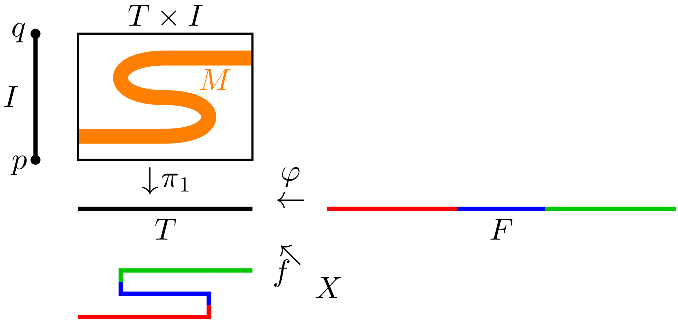

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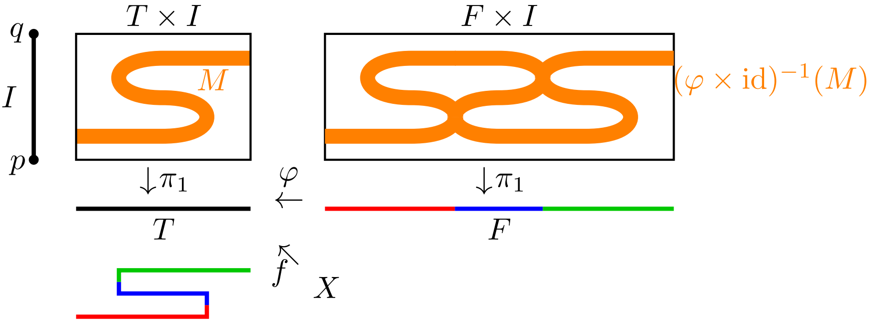

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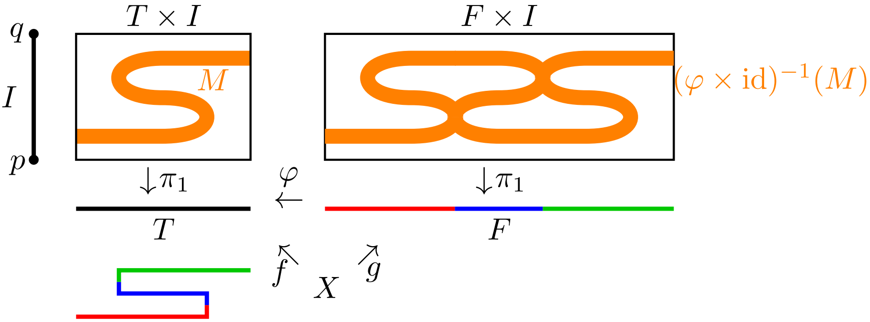

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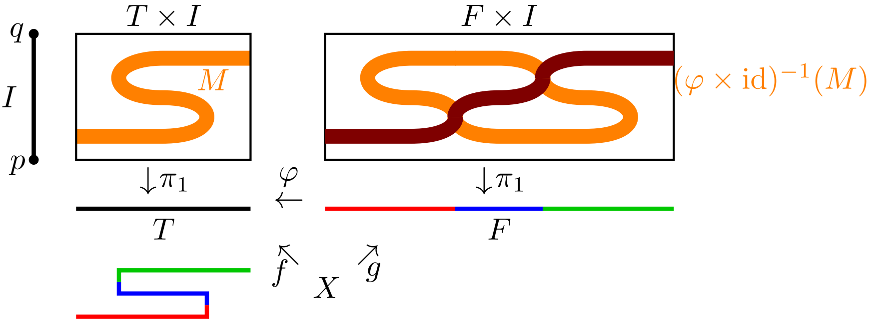

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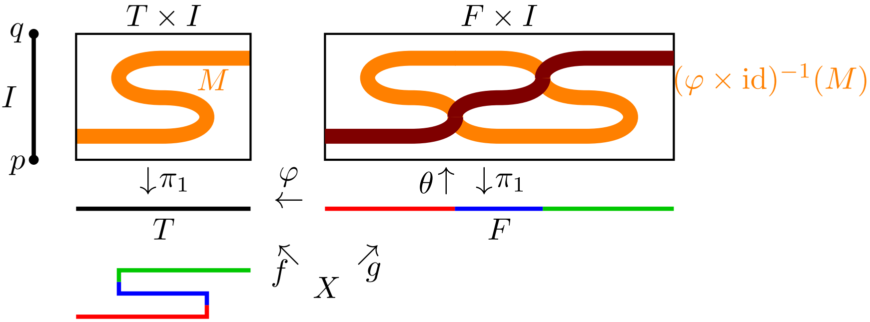

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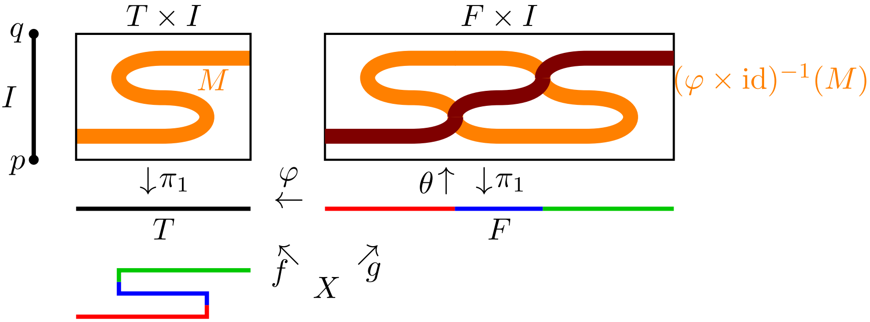

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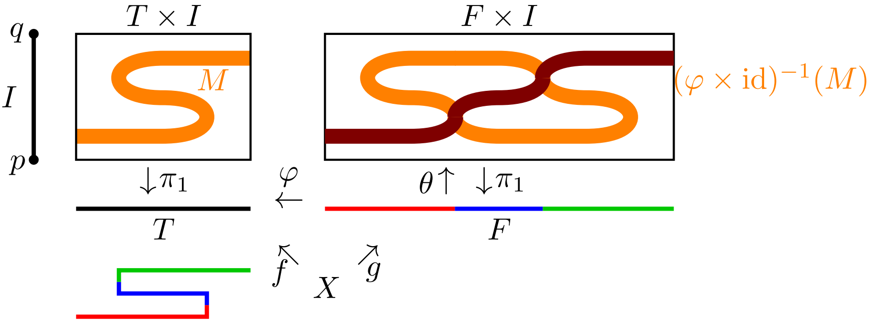

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Idea: Find sequence of simple folds $T \leftarrow F_1 \leftarrow F_2 \leftarrow \cdots \leftarrow F_n$ such that in F_n, separator has a subset S' such that π_1 maps S' one-to-one onto F_n.

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(Informal definition)

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Theorem

Given any separator $M \subseteq G \times (0,1)$ and any open set $U \subseteq G \times (0,1)$ with $M \subseteq U$, there exists a separator $S \subset U$ with a stairwell structure of odd height.

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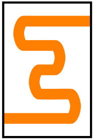

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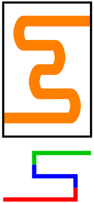

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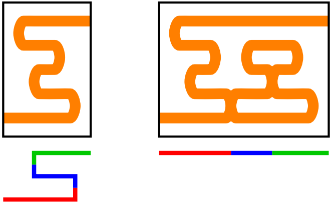

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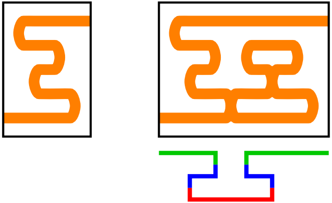

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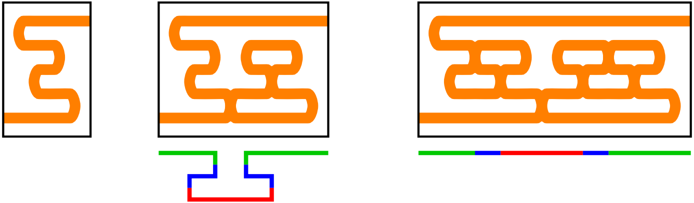

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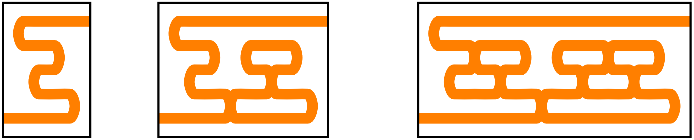

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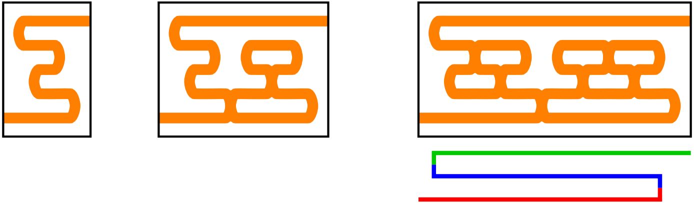

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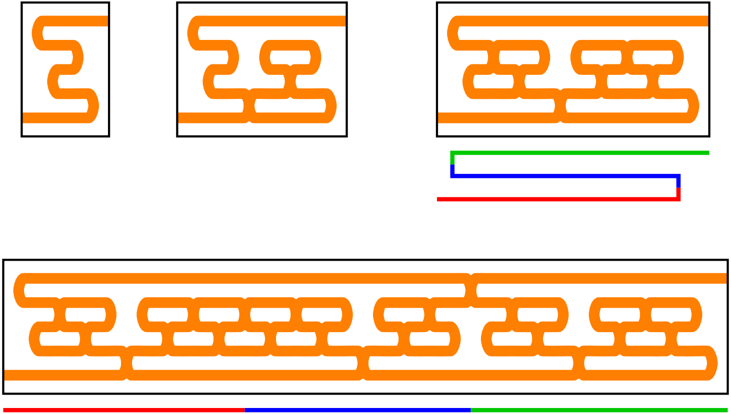

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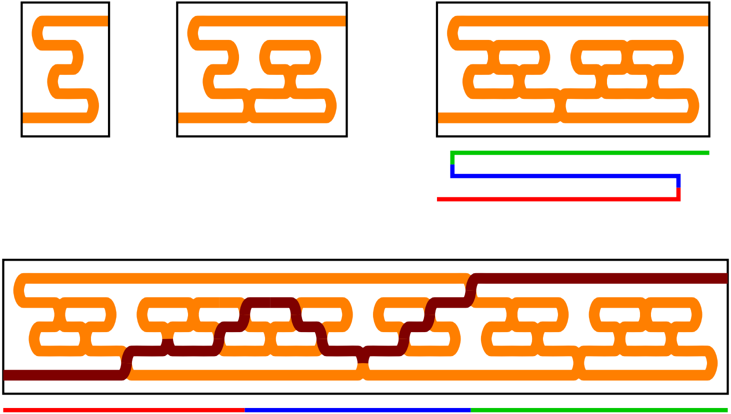

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